Early Mathematical Development in Sociocultural Context

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Mathematical Development in Early Childhood

• Research has found that mathematical thinking begins during infancy and undergoes extensive development over the first five years of life.

• It is just as natural for young children to think mathematically about their world as it is for them to use language.

• Young children develop informal mathematical knowledge that is based on the manipulation or representation of concrete objects. It is less abstract than the formal mathematical knowledge of school-age children.
Examples of Informal Mathematical Knowledge

• Counting sets of objects

• Using one-to-one correspondence to construct or compare sets

• Solving simple arithmetic problems with sets of visible or hidden objects

• Naming and analyzing features of 2-and 3-dimensional shapes

• Representing the location and movement of objects in space

• Identifying and duplicating simple patterns

• Measuring objects by direct comparison of length or weight
Why is informal mathematical knowledge important?

1. It serves as a foundation for the development of formal mathematical knowledge in elementary school

\[ 2 + 2 = 4 \]
\[ OO + OO = OOOO \]
Why is **informal mathematical knowledge** important?

2. Mathematics knowledge at school entry is the strongest predictor of later achievement yet identified

**Meta-Analysis of Research on School Readiness and Later Achievement by Duncan et al. (2007)**

- An analysis of 6 longitudinal data sets relating school-entry skills to teacher ratings and test scores of reading and math achievement
## Predictors of Later Learning

<table>
<thead>
<tr>
<th>School Entry Skill Domain</th>
<th>Effect Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>.34</td>
</tr>
<tr>
<td>Language &amp; reading</td>
<td>.17</td>
</tr>
<tr>
<td>Attention</td>
<td>.10</td>
</tr>
<tr>
<td>Socioemotional</td>
<td>.00</td>
</tr>
</tbody>
</table>
The Early Development of Mathematical Cognition in Socioeconomic and Cultural Contexts

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Supported by the Interagency Education Research Initiative (grant no. 9979974)
Mathematical Knowledge of American, Chinese, and Japanese Children at Age 4 Years
Socioeconomic Variation in Informal Math Knowledge

- A growing body of research has revealed a socioeconomic gap in young children’s mathematical knowledge.

- Mathematical knowledge is less extensive in low-income pre-kindergarten children than in their middle-income peers.

- The socioeconomic gap is broad, and it emerges as early as 3 years of age.

- Thus, children from different sociocultural backgrounds enter elementary school at different levels of readiness for school mathematics.
Math Knowledge in American 4-Year-Olds

Mean Percent Correct

Low-income
Middle-income

Number
Arithmetic
Geometry
Measurement
Patterns
Composite
The Math Gap

• The SES-related gap in early mathematical knowledge is present by age 3 years and widens during early childhood in the United States but narrows in China
CMA Scores of American Children

- Lower SES
- Higher SES

Age

3.0 3.9 4.0 4.9
CMA Scores of Chinese Children

Lower SES vs Higher SES

Age

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

3,0 3,9 4,0 4,9
Why does the math gap widen in the United States?

American preschool children from low-income families, in comparison with their middle-class peers, receive less support for mathematical development in their home and preschool environments.

The Home Learning Environment of Young, Economically Disadvantaged Children:
- Fewer math-related materials
- Fewer math activities in which an adult participated
- Lower parental expectations about mathematical development prior to elementary school
Why does the math gap narrow in China?

Possible explanations:

- Universal preschool (in urban areas)
- National mathematics curriculum for preschools
- High parental and teacher expectations for mathematical development
“Which mathematical abilities or skills do children typically develop by the end of preschool?”

<table>
<thead>
<tr>
<th>Abilities Within the Developmental Range of Pre-K Children</th>
<th>Parents</th>
<th>Teachers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>American</td>
<td>Chinese</td>
</tr>
<tr>
<td></td>
<td>Low-SES</td>
<td>Mid-SES</td>
</tr>
<tr>
<td>Count a row of 10 objects</td>
<td>89%</td>
<td>98%</td>
</tr>
<tr>
<td>Show which doll in a row of 5 is second</td>
<td>71%</td>
<td>92%</td>
</tr>
<tr>
<td>Solve small addition or subtraction problems presented with objects</td>
<td>57%</td>
<td>84%</td>
</tr>
<tr>
<td>Share 12 crackers equally among 3 friends</td>
<td>48%</td>
<td>61%</td>
</tr>
<tr>
<td>Make a simple pattern with colored beads</td>
<td>86%</td>
<td>98%</td>
</tr>
<tr>
<td>Match 2 identical triangles in different orientations</td>
<td>70%</td>
<td>82%</td>
</tr>
</tbody>
</table>
US-Low  US-Mid  Japan  China
SES      SES
Curricula in widespread use in public preschool programs have been found to be ineffective, relative to controls, at enhancing children’s mathematical knowledge:

- The Head Start Impact Study (2005, 2008) found no difference in mathematical knowledge between intent-to-treat intervention (Head Start) and control children at the end of the pre-kindergarten year.

- The IES Preschool Curriculum Evaluation Research Initiative (2008) found that curricula in widespread use in public preschool programs are not effective in the area of mathematics relative to control curricula. Math-focused curricula, however, can be effective.
Preschool Curriculum Evaluation Research Project: Randomized Trial of a Pre-K Mathematics Intervention

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The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education through Grant R305J020026 to UC, Berkeley. The opinions expressed are those of the authors and do not represent views of the U.S. Department of Education
Objectives

• Implement a pre-kindergarten mathematics curriculum in preschools serving low-income children in California and New York using randomly assigned classrooms/teachers

• Evaluate the effects of the curriculum on children’s early mathematical development and their learning environments (home and classroom)

• Compare child outcomes in teachers’ first and second years of implementation
Design

• 40 classrooms: 10 Head Start, 10 state-funded preschools per state (California and New York)
• Random assignment of classrooms to intervention or control conditions, using block randomization at the program level
• Random selection of 8 low-income children per classroom
• Child sample:
  Cohort 1 (Year 1): 316 pre-k children
  Cohort 2 (Year 2): 312 pre-k children
Components of the Intervention

*Pre-K Mathematics Curriculum* by Klein & Starkey

**Classroom component:**
- 29 teacher-guided small group activities with manipulatives
- Activities are organized in units containing closely related mathematical content
- Each activity requires that teachers make developmental adjustments for individual children and use key mathematical language
- Teachers keep written records of each child’s learning or difficulties experienced during the activity
- **Supplementary mathematics software:** 27 computer-based activities, for classroom use, from *DLM Express* by Clements & Sarama
- **Math learning center**

**Home Component:**
- 18 activities for home use by parent-child dyads (in English and Spanish)
Research Measures

• Fidelity of Implementation of *Pre-K Mathematics*

• *Early Mathematics Classroom Observation* (EMCO) of teachers’ mathematics practices

• *Parent Survey* of use of *Pre-K Mathematics* home activities

• *Child Math Assessment (CMA)* of children’s mathematical development
Child Math Assessment (CMA)

Assesses informal mathematical knowledge across several conceptual areas of mathematics

- Number
- Arithmetic operations
- Space/Geometry
- Pattern knowledge
- Measurement
Professional Development of Intervention Teachers

Year 1:
- Workshops (4 days Fall and 4 days Winter)
- On-site facilitation (fidelity with feedback during classroom visits every 2 weeks)

Year 2:
- Refresher workshops (2 days Fall and 2 days Winter)
- On-site facilitation (fidelity with feedback during classroom visits every month)
Overall Fidelity During Years 1 and 2 of Implementation

<table>
<thead>
<tr>
<th>Curriculum Component</th>
<th>Year 1</th>
<th>Year 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small Group Activities</td>
<td>.89</td>
<td>.96</td>
</tr>
</tbody>
</table>
Principal Hypotheses

• Change in Classroom Practices: More math support will be provided by intervention teachers than by control teachers

• Impact on Children’s Mathematical Knowledge: More extensive mathematical knowledge will develop in intervention children than in control children
Effects of the Intervention on Classroom Mathematics Practices

• Early Mathematics Classroom Observation (EMCO) instrument measures amount of time teachers spend on focal (intentional) math activities per child

**Hypothesis:** The curricular intervention will increase the amount of focal math provided by intervention teachers
Findings on Classroom Math Practices

- Significantly more minutes of **focal** (intentional) math per child were provided by intervention teachers than by control teachers, $p<.0001$ (Cohort 1) and $p<.0001$ (Cohort 2)

- Significantly more minutes of **focal** math per child were provided by intervention teachers on small-group treatment days than on no-small-group treatment days, $p<.02$ (Cohort 1) and $p<.001$ (Cohort 2)
Amount of Focal Math Provided by Intervention and Control Teachers

Minutes of Math Support Per Child Per Day

Year of Intervention

Year 1
- I-Treatment Days: 22
- I-Non-Treatment Days: 7
- Control: 7

Year 2
- I-Treatment Days: 15
- I-Non-Treatment Days: 6
- Control: 3
Research Question: Does the number of minutes of focal math that teachers provide predict change in children’s mathematical knowledge?

Findings:
• Amount of focal math significantly predicted change in intervention children’s CMA scores, $F(1,17)=15.32$, $p<.0001$ (Cohort 1); $F(1, 17)=6.36$, $p<.025$
Effects of the Intervention on Children’s Mathematical Development

**Hypothesis:** The curricular intervention will have a positive impact on children’s mathematical development.

**Findings:**
- Intervention and control groups did not differ on their CMA scores at pre-test, but they did differ significantly at post-test, $p<.0001$ (Cohort 1) and $p<.0001$ (Cohort 2).
CMA Scores of Intervention and Control Children in Fall and Spring

Time of Assessment

Fall (Pretest)  Spring (Posttest)

Intervention Cohort 1
Control Cohort 1
Intervention Cohort 2
Control Cohort 2
•**Cohort 1 effect size** (Cohen’s $d$)=.58 (What Works Clearinghouse), a 62% increase in math knowledge for intervention children relative to control children

•**Cohort 2 effect size**=.70 (What Works Clearinghouse), a 79% increase for intervention children relative to control children

**Conclusion:**
The findings from both cohorts support the principal hypothesis of a causal relationship between the curricular intervention and children’s mathematical development.

Scaling Up the Pre-K Mathematics Intervention:  
A Randomized Controlled Trial

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The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education through Grant R305K05186 to UC, Berkeley. The opinions expressed are those of the authors and do not represent views of the U.S. Department of Education
Design

- A cluster randomized trial with preschool site as the unit of randomization
- Classrooms were randomly selected within sites
- Pre-kindergarten children were randomly selected within classrooms (8-10 per classroom)

**States:** California and Kentucky/Indiana

**Program Type:** Head Start and State-Funded Pre-Kindergartens

**Treatment Condition:** Intervention (Pre-K Mathematics curriculum) vs. Control (Business-as-usual/Creative Curriculum)

**California and Kentucky/Indiana**

**Preschool Sites:**
- CA = 36 (18 I and 18 C)
- KY = 26 (13 I and 13 C)

**Classrooms:**
- CA = 48 (24 I and 24 C)
- KY = 46 (24 I and 22 C)
Significant Findings from the Child Math Assessment

- **Condition:** $F(1,666) = 11.48, \ p<.0001$
- **Time:** $F(1,666) = 1740.11, \ p<.0001$
- **Gender:** Females had significantly higher scores on the CMA than males regardless of treatment condition, $F(1,666) = 5.45, \ p<.02$
- **Condition X Time:** Intervention and control children did not differ on the CMA scores at the pretest, but Intervention children had significantly higher CMA scores at the posttest, $F(1,666) = 171.28, \ p<.0001$
- Intervention children’s gains in mathematical knowledge over the pre-k year was approximately twice that of control children
- **Effect Size = .83**
What Was NOT Significant:

- State (p=.15) and State X Time (p=.54)
- Program Type (p=.16) or Program Type X Time (p = .74)

Thus, the effectiveness of the intervention is robust across varied contexts: California Head Start and state preschool programs serving ethnically diverse urban families, and Kentucky/Indiana Head Start and state preschool programs serving predominantly white rural families
CMA Scores of Intervention and Control Children in Fall and Spring
TEMA-3 Scores of Intervention and Control Children in Fall and Spring
In closing, intervention research has shown that it is possible to enhance pre-kindergarten children’s mathematical knowledge through the use of an effective curricular intervention.

Furthermore, it is possible to obtain these effects when the intervention is scaled up and implemented in geographically and ethnically diverse contexts.